Time Series Analysis of Complex Dynamics in Physiology and Medicine

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1 Introduction

A hallmark of living organisms is that they are not constant in time. Subcellular, cellular, and supercellular processes such as the cycle of cell growth and division, voltage fluctuations in excitable cell membranes, respiration, bood pressure regulation, and the sleep-wake cycle provide spectacular examples of complex rhythms. Interest in these complex fluctuations has been stimulated in recent years by the widespread recognition that deterministic dynamical systems can display *chaotic dynamics*: aperiodic rhythms sensitive to the initial condition [11, 55, 13, 23, 1, 47]. Although analyses of theoretical models and experiments in controlled situations have provided good evidence that chaos can sometimes be found in biological systems [29, 2, 34, 10, 30], chaos itself serves more often as a motivating idea for research than an unequivocal scientific finding.

In this article we will provide a brief critical summary of time series analysis in biology with emphasis on physiology and medicine. We will first briefly summarize standard methods of time series analysis and indicate how these have been applied. Then we will indicate some of the ways in which concepts from nonlinear dynamics are being applied. Since our strongest interest is in human physiology, most examples will be drawn from this area. However, time series analysis appears in so many different ways in various fields and subfields that we do not attempt a comprehensive review, but give illustrative examples. We also do not discuss the development of theoretical models for biological systems since other sources provide detailed information about these areas [23].

2 Standard methods of time series analysis

The most basic sort of time series analysis is carried out by the human eye, often assisted with calipers to measure distances on a paper chart (corresponding to time intervals in the original data). This is not as primitive as it sounds. The human eye is an excellent pattern recognition device and is capable of carrying out the sophisticated analyses needed to classify time series. For example, in the field of electrocardiography, the interpretation of even exceedingly complex electrocardiograms (ECGs) as carried out, for example, with virtuousity by Pick and Langendorf [63], requires nothing more than application of several basic concepts in cardiology combined with measurement of timing of the occurrence of beats on comparatively short records. Interpretation of electroencephalograms (EEGs) is carried out in similar fashion by skilled clinicians who have learned how to interpret the frequency, amplitude and morphology of recordings of electrical activity from different scalp locations [61]. This is sufficient for the identification of a great number of different clinical disorders.

Computer analysis of time series can provide routine diagnosis, such as reading ECGs or EEGs, or can carrry out tasks such as the detection of heart beats (or the lack thereof) in implantable pacemakers or defibrillators. Most medical instrumentation involves signal processing of some sort.

In research, quantitative analysis of physiological time series starts (and often ends) with an analysis of the mean and standard deviation. In some cases, these simple statistics can provide information of physiological importance. For example, the mean heart rate (over 10s of seconds) can be used to indicate a level of exertion, and a low standard deviation of heart rate has been shown to be associated with pathology [46]. In respiration, the frequency and the duration of the inspiratory and expiratory phases change with age and are different in different species [18, 58].

The standard deviation by itself often does not provide an adequate characterization of *fluctuations* in physiological systems. For the purpose of characterizing fluctuations, the power-spectrum and autocorrelation function, and transfer functions, have successfully been applied. These techniques were introduced to physiology by skilled workers with a background in engineering and have seen significant applications to physiology over the past 30 years. Power spectra and allied techniques have been used in various fields in which the frequencies of oscillations are believed to have functional or clinical significance such as heart rate variability [45, 4], tremor [6], electroencephalography [50, 56]. Perhaps because of the systems engineering background of most workers studying spectral analysis, different frequencies are usually associated with different mechanisms that lead to superimposed oscillations.

3 Time series analyses using methods introduced from nonlinear dynamics

Recognition of the importance of nonlinear phenomena in physiological systems has a long history — its beginnings are perhaps represented by the work of van der Pol and van der Mark [76] in the early part of the century. In Nobel Prize winning work, Hodgkin and Huxley [35] related the dynamics of excitable cell membranes to a system of coupled nonlinear differential equations.

The recent realization that nonlinear dynamical systems can display deterministic chaos has had a strong impact on research in time series analysis in physiology and medicine [23]. Whereas a generation ago, researchers were delighted to conjecture that a complex time series from a neuron was well-described in terms of random walks [20], there is now a strong inclination to interpret physiological time series in terms of chaos. Often, this interpretation involves the use of time series analysis techniques motivated by chaotic dynamical systems. In this section we briefly review several of the main concepts in nonlinear dynamics that have been applied to time series analysis.

Bifurcations

One of the most basic concepts in nonlinear dynamics is *bifurcation*. A bifurcation is a change in the qualitative features of the dynamics that arises as some parameter describing the systems changes. Bifurcations may be associated with the onset or the annihilation of oscillations, a sudden change in the period of an oscillation, or the onset or annihilation of chaotic dynamics.

Figure 1 shows an example of bifurcations in a theoretical model of a multi-looped feedback control system represented as a delay differential equation [53]. Increasing the gain of the control function leads to a cascade of period-doubling bifurcations and, eventually, an aperiodic chaotic rhythm. Examples such as this in model systems abound. It is rarer to find good examples of bifurcations in experimental systems. One experimental system that does show similar phenomena is periodically stimulated chick heart cells. Figure 2 shows a series of traces displaying period-doubling bifurcations and also an aperiodic chaotic rhythm [29].

The occurrence of complex bifurcations is well known in medicine. For example, in cardiology complex changes of rhythm associated with various arrhythmias are well documented and in some cases may be associated with bifurcations in nonlinear dynamical equations [22]. For example, the appearance of alternans rhythms in which there is a beat to beat alternation of ECG waveforms may in some cases be associated with period doubling bifurcations [28, 73].

Time series analysis techniques to detect and characterize bifurcations have not been widely developed. Smith *et al.* [73] have proposed an FFT-based statistic for quantifying alternation and a method for detecting and quantifying

Figure 1. Time series generated from equations modeling a multi-loop negative feedback system showing bifurcations that occur as the gain of feedback is increased. (Reproduced from [53].)

(a) Periodic orbit

(b) Period-2 orbit

(c) Period-4 orbit

(d) Chaotic orbit

Figure 2. Phase plane embedding (a) and Poincare map (b) for the time series in Fig. 1d. Successive values, designated P_i, of $P(t-2.01)$ for crossing $P(t) = 0.55$ with $dP/dt > 0$ are determined from the data in (a). (Reproduced from [53].)

alternation that uses an embedding-space formulation is given by Kaplan [41]. In many circumstances, clear examples of bifurcations are difficult to document and a variety of additional measures have been to developed to characterize complex time series.

Dynamical representations of time series

Although it is most common to plot time series as a function of time a variety of other methods suggested by nonlinear dynamics provide powerful insights into dynamics in some circumstances, as illustrated in Figures 1 and 2.

One technique, *phase plane embedding*, involves plotting $x(t+\tau)$ vs. $x(t)$, giving a trajectory of the time space. Plots of a variable as a function of its delayed value were first used in theoretical studies of chaos in physiological systems modeled by delay differential equations [21], and this technique has since been used widely for systems in which only one variable is easily measured [69]. Embeddings of a time series in higher dimensional phase spaces by plotting the current value as a function of several time lagged values can be easily implemented for computational purposes.

An example of a phase plane embedding for the time series in Fig. 1d is shown in Fig. 2a. In this plot, the trajectory is well confined in a limited region of phase space. Further insight into the dynamics in this example can be obtained by examining the flow on a cross-section to the trajectory of the flow. Successive returns to a cross-section to the flow are plotted in Fig. 2b. The points fall approximately on a one-dimensional single humped curve known to give rise to chaotic dynamics. The map, which gives successive returns to the cross section is usually called the *return map* or *Poincaré map*. A recent example in which an unstable cardiac preparation is perturbed and stabilized by feedback calculated from a Poincaré map is described in [52].

In cases where analysis of the time series suggests a one-dimensional map, it may be possible to derive a form for the map on a theoretical analysis of the underlying mathematical problem. An example is provided by the periodically stimulated heart cell aggregates, Fig. 4. Theoretical analysis [29, 30] of the effects of periodic stimulation of a limit cycle oscillation, show that (i) if there is rapid relaxation to the limit cycle and (ii) if the stimulus does not alter the properties of the oscillator then a plot of successive phases of stimulus in the limit cycle follows a one-dimensional map. A map derived from the time series for Fig. 2a once again falls on a one-dimensional single humped curve.

In Figs. 3 and 4, the time embedding techniques help to identify the underlying deterministic dynamics governing the time evolution. Although such techniques can be readily tested on data sets, there is no guarantee that simple one (or higher) dimensional maps will be identified. To give an idea of the difficulties that arise in practical situations consider data set B. Figure 5 shows several two-dimensional embeddings in the data in which the heart rate is displayed as a function of its time lagged value. Panels 5 (a),(b), and (c)

Figure 3. Time series showing the effects of periodic stimulation (sharp spikes) on spontaneously beating aggregates of embryonic chick heart cells. (Reproduced from [29] and [30].)

(a) Transition from 1:1 phase locking to 2:2 phase locking.

(b) 4:4 phase locking.

(c) Irregular dynamics reflecting deterministic chaos.

(d) Return map showing the phase of stimulus $i+1$ as a function of stimulus i.

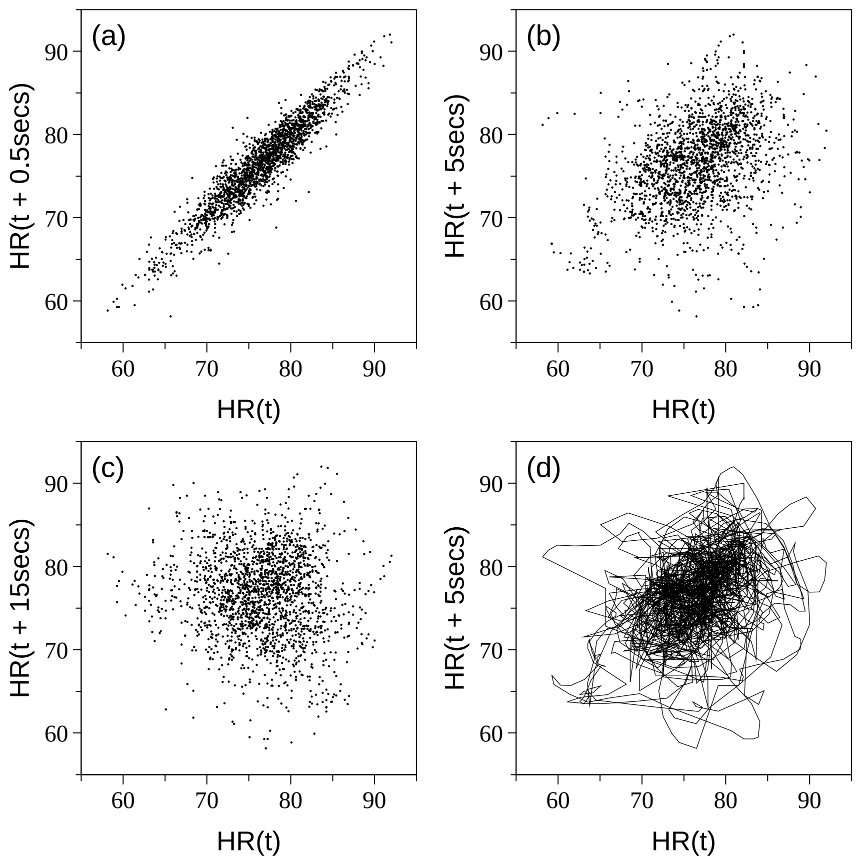

Figure 5. Embedding plots of the heart rate time series from Data Set B, with different embedding lag τ.

(a) $\tau = 0.5$ secs.

(b) $\tau = 5$ secs.

(c) $\tau = 15$ secs.

(d) $\tau = 5$ secs (as in (b)), but now consecutive points have been connected. This shows that the filament structures near the periphery in (b) are the result of single passes of the trajectory through sparsely populated areas.

show values stepping through the data set at 0.5 sec increments, and panel 5(d) shows the embedded trajectory based on Fig. 5(b). The ellipsoidal structure in 5(a),(b) simply reflects the autocorrelation of the heart rate for small time delays — even random numbers that have been low-pass filtered to produce such correlations will show this structure. Depending on the embedding delay τ, other forms of structure can appear in the embedding. Often, this structure is consistent with passing random white noise through a linear coloring filter. None of these plots give an indication of structure present in the data that would indicate a deterministic origin. Of course, one cannot exclude the possibility that some higher dimensional embedding would disentangle the spaghetti mess in Fig. 5(d). A major problem is to decide what is a proper embedding dimension. The "false neighbors" technique proposed by Kennel *et al.* [44] addresses this issue.

Two-dimensional representations of physiological variables have provided a useful tool for plotting physiological data of periodic rhythms. For example, phase plane plots of the volume and flow during a breath provide a representation of the respiratory cycle [59]. In motor control, there have been descriptions of limb position using phase plane plots [3].

Dimension and Lyapunov Numbers

A number of measures of complex time series have been developed based on concepts from nonlinear dynamics. Although such measures have well-defined meanings in idealized situations, in practice, the complex nature of physiological time series often makes interpretation of these measures difficult if not impossible. We briefly review several methods currently being employed. An excellent, detailed review of these techniques along with the pitfalls is [27].

The *dimension* [17, 26] gives a statistical measure of the geometry of the cloud of points. In deterministic chaotic systems the dimension is frequently (but not always!) a fractional number and is independent of the embedding dimension m when m is large enough. In practice, with physiological data, there are many difficulties involved in the analysis of a system using dimension. Some of the issues involved include the stationarity of the dynamics, noise, the sampling rate of the time series, the need to use finite length scales imposed by the finite size of data sets, and selecting appropriate convergence criteria to assert the existence of a well-defined dimension. To deal with some of these problems, computation of the local or "pointwise" dimension has been suggested. [54, 33, 19]. For example, Skinner *et al.* have proposed a technique in which a separate estimate of the correlation dimension is calculated for each point in the time series, based on the several nearest neighbors in the embedding space [72, 49].

Another set of statistics in wide use are *Lyapunov exponents*. These measure the average local rate of divergence of neighboring trajectories in phase-space embeddings [77]. If a system is known to be deterministic, a positive Lyapunov

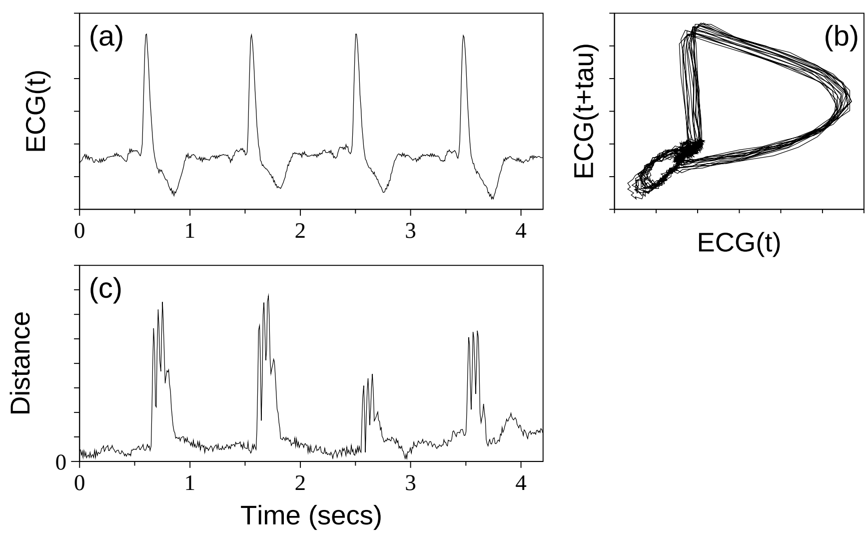

Figure 6.

(a) A surface electrocardiogram (ECG). The tall, upward spikes (the "QRS complexes") correspond to ventricular activation, the more rounded downward excursions correspond to ventricular relaxation (the "T-wave").

(b) A embedding of the ECG signal. $m = 2$, $\tau = 50$msecs.

(c) The distance between two segments in the embedded ECG trajectory (with $m = 3$, $\tau = 50$msecs). Initially the segments are quite close, but are rapidly separated during the QRS complexes. Lyapunov exponent calculations that assume that trajectories will separate in an approximately exponential fashion may be mislead by data like this.

number can be taken as a definition of a chaotic system. There are now a variety of algorithms for estimating Lyapunov exponents from time series. An early algorithm in wide use developed by Wolf and colleagues [77] estimates the divergence of pairs of neighboring trajectories. Unfortunately the Wolf algorithm has some serious limitations in the analysis of biological data, but this is still not always recognized [14]. One problem arises when there are large derivatives of approximately periodic functions. An example of an ECG and its embedding in 2 dimensions is shown in Fig. 6. Figure 6(c) is a plot of the distance between 2 nearby points (in a 3-dimensional embedding). The Wolf algorithm would give positive contributions to the Lyapunov number associated with the large increases in the distance in Fig. 1(c), but this does not reflect an exponential divergence of the trajectories.

Other algorithms for estimating Lyapunov exponents fit various forms of functions to the embedded data and calculate the local divergence based on local linearization of these functions [68, 62, 78, 7]. Critical issues in the reliability of the estimation of Lyapunov exponents are the influence of noise, the choice of fitted functions, the size of the neighborhood over which the fitting is done, and introduction of spurious exponents from use of too high an embedding dimension. There has not been a convincing demonstration that current techniques are useful for looking at systems whose attractors are high-dimensional $m > 3$.

Surrogate data and tests for determinism

The above measures are frequently difficult to interpret since stochastic systems may also yield a value for the dimension and Lyapunov number. Recent work has emphasized the need to compare the values of these statistics as generated from a time series, to the values generated by suitably constructed stochastic processes. The structure that a statistic is detecting in the cloud of embedded points may possibly be the result of a non-chaotic dynamical mechanism — for example stochastically forced linear dynamics. We have found, for example, that heart rate signals cannot systematically be distinguished from stochastically forced linear dynamical systems [38]. The same is true for the EEGs we have studied [24].

Theiler *et al.* [75] have proposed a form of bootstrapping that enables any statistic — even those not typically associated with chaos — to be used potentially as a probe of nonlinearity in the data. The bootstrapping involves the generation of "surrogate data" that has certain properties in common with the test data. For example, random surrogate data can be synthesized that has the same autocorrelation function as the test data. The surrogate data can be used informally to evaluate whether a finding of low-dimensional dynamics indicates chaos (see, for example, a study of ventricular fibrillation using a dimension statistic [42]).

The difficulties above indicate the need for having measures that address whether a given time series is generated by a deterministic process. Our work

[37, 38, 39] is based on the observation that deterministic systems will have well-defined vector fields. By embedding a time series using standard methods and examining the flow in coarse-grained hyperboxes we can study the extent to which flows through local neighborhoods are locally parallel as would be found for deterministic systems. This work, (see also the article by Kaplan in this volume) has shown that several short time series showing heart rate variability or electroencephalograms did not show evidence for determinism but were indistinguishable from a stochastically forced linear system with the same power spectrum [38].

Another approach to the analysis of determinism is to distinguish between deterministic chaotic systems and stochastic systems based on the extent to which future values of the system can be forecast from past values i.e. to *test the predictability*.

There are two complementary approaches to using predictability. The falloff of predictability with increasingly future forecasts can sometimes be used to distinguish different types of dynamics. Although it is sometimes possible to make short-term forecasts for chaotic systems, the predictability is expected to fall off on a time scale governed by the positive Lyapunov exponents. For example, Sugihara and May examined month-by-month records of the numbers of measles and chicken-pox patients, and showed that the measles record was more predictable than white noise, and that the predictability falls off in a manner consistent with chaos [74]. The interpretation of results such as these is somewhat difficult, since stochastically forced linear systems can show similar fall-offs.

A second approach, due originally to Casdagli [9] examines whether locally fit linear models perform better at forecasting than globally fit models. If the locally fit models are better, this provides evidence for nonlinear structure in the dynamics — even nonlinear stochastic structure can be detected in this manner. This approach has been adopted, for example, by Longtin [51] in the analysis of spike trains from sensory neurons.

4 Difficulties in Analyzing Physiological Data

In this section we consider in more detail some of the properties of physiological data that impose difficulties in the use of many nonlinear dynamics statistics.

Nonstationarity

If statistical characterizations of a time series are not constant in time the time series is *nonstationary*. One cause of nonstationarity in time series is the constantly changing environment. An obvious but important example is found in the study of 24-hour heart rate variability — in a natural setting, subjects are constantly changing their posture or level of activity. Even when sleeping, changes in sleep stage have a demonstrable effect on heart rate variability.

Other forms of nonstationarity may arise from transients. One role of the various physiological control systems is to respond to changes in environment or other "perturbations." Usually, a physiological control system is not at steady state.

In some cases, the interaction between control systems may lead to very long transients — for example, the time scales over which short term cardiovascular control systems act ranges from seconds (the parasympathetic nervous system) to hours (renal fluid volume) [32]. Altogether, variability in physiological parameters often has a $1/f$ spectral form, suggesting strong nonstationarity of even long-term physiological recordings [60, 43].

Several approaches have been used to deal with nonstationarity in physiological time series. The simplest is to use short time series where, it is presumed, nonstationarity is not a severe impediment (e.g., [42]). Another technique is to attempt to correct for slow drifts, either by the subtraction of trends or taking the first difference of the time series (e.g., [74]). The first step is obviously to detect the existence of nonstationarity — visual clues to nonstationarity can be provided by recurrence plots [16], where the time at which the embedded trajectory returns closest to itself are indicated for each point in the trajectory.

Noise

Physiological systems often display complex fluctuations that are frequently identified with stochastic "noise". Most physiological processes ultimately are affected by the opening and closing of subcellular ion channels, and most workers believe that the kinetics of the channels is best decribed by stochastic processes. Consequently, stochastic processes or noise are ubiquitous in living systems. Yet conventional wisdom is that the averaging that occurs in going from one hierarchical level to the next, can lead to deterministic models (e.g., the Hodgkin-Huxley equations) being appropriate for cellular and supercellular processes (see Fig. 2). There is not now a good mathematical understanding of the properties of nonlinear dynamical systems in the presence of noise, or how the various statistical measures discussed in the previous section are affected by noise.

In addition to this problem, many physiological processes display non-gaussian noise, and the statistics can be dominated by "outliers". One example, from heart rate variability, concerns the existence of premature ventricular beats. The premature beats introduce very short inter-beat intervals into the heart rate record which may have a relatively fixed relationship with the timing of the preceeding and following beats.

An anecdote may illustrate the potential role of such artefacts in nonlinear analysis of physiological time series: in a study of changes in heart rate variability with aging, a comparison was done between the correlation dimension of heart rate and the dimension of a random surrogate time series with the same power spectrum. The purpose was to see if the correlation dimension found evidence for nonlinear dynamics in the heart rate time series. The results were extremely strong: in young people (mean age 28 years) there was not difference between the heart rate time series and the surrogate data. In old people (mean age 75) there was a very distinct and systematic difference: the dimension of the heart rate time series was much less than the dimension of the surrogate data. A careful investigation of the reasons for this revealed that premature beats (which have a different physiological mechanism than normal beats) were producing spikes in the heart rate record and the randomization technique used in constructing the surrogate data was transforming these highly-ordered spikes into random white noise. Although the correlation dimension calculation was not sensitive to the occasional spikes in the heart rate record, it was very sensitive to the white noise in the surrogate data. Since premature beats occur more frequently in old people (something that is well known clinically), the correlation dimension was able to distinguish between heart rate and the surrogate data only in the old people. When the premature beats were removed from the analysis, the difference between heart rate and the surrogate data disappeared.

High dimensions

Physiological systems are typically high dimensional and as such are difficult to analyze. For example, Data set B in the Santa Fe Time Series Competition consists of simultaneous measurements of heart rate, respiration force, and blood oxygen concentration. (See [?], this volume.) These were only a part of the original data set, which included systolic and diastolic blood pressure. When one realizes that each of these measurements is itself the end result of dynamics coupled through nonlinear delayed feedbacks, the possibility emerges of high-dimensional dynamics. Perhaps the intrinsic difficulty of these problems is reflected in the reluctance of most participants in the competition to analyze Data Set B, in comparison to some of the other sets.

Although it is theoretically possible under certain conditions in deterministic systems for a single measured variable to be able to represent the entire system's dynamics, it is not well understood when this is a practical approach — the relationship between, say, respiration force and heart rate is sufficiently complex that it may be necessary to include both measured signals (as well as other coupled signals such as blood pressure) in a dynamical analysis of cardiovascular control. The use of multiple signals can lead to very high embedding dimensions (each signal may need to have several lags used in order to get a meaningful representation). The use of most nonlinear dynamics techniques in high-dimensional embeddings has not been well studied, and there is little knowledge about the best ways of representing multiple signals or of identifying interactions or coupling among signals.

As another example, consider the spread of excitation in heart muscle. This is a problem which is intrinsically infinite dimensional. Common measurement methods involving surface ECG recordings reflect a projection of this problem to low dimensions. However, the ECG from a single lead is at best a crude indicator of the three dimensional spread of excitation, particularly for rhythms with complex spatial organization such as ventricular fibrillation and ventricular tachycardia. Current mapping cardiac electrical activity are being made with 512 simultaneous electrodes [36] and optical techniques provide resolutions of upwards of 10000 pixels [12]. How best to reduce such measurements to a low-dimensional representation is an unsolved problem.

Related issues involve the EEG as a measure of brain activity. Surface EEG recordings reflect averages of electrical activity over millions of cells. The functional significance of the EEG waves are not well understood. Claims that this data reflects low dimensional dynamics have been numerous, but our own preliminary analysis of this problem does not show low dimensional dynamics or evidence for determinism in data sets of normal EEG activity.

5 Applications of Time Series Analysis – Chaos or Characterization?

One of the principal goals to date of nonlinear dynamical time series analysis in physiology and medicine has been to establish whether time series arise from chaotic dynamical systems. There has been an unfortunate tendency to assume that if a computer program prints out a finite correlation dimension or one that has a non-integer value, or the program gives an estimate of a maximum Lyapunov exponent that is positive, then the time series is chaotic.

The search for deterministic chaos in complex physiological systems has focussed discussion away from other important issues. One possibility is that nonlinear statistics such as the correlation dimension can be effective ways of describing time series from physiological systems *even though these systems may not be chaotic.* Particularly in the EEG literature, there has been an attempt to use the correlation dimension to distinguish between different mental or physiological states [5, 66]. There have also been attempts to characterize the "complexity" of heart rate and blood pressure variability using statistics motivated by dimension and entropy [40, 65].

Insofar as it is desired to use statistics such as the dimension or entropy as characterizations of the (perhaps stochastic) dynamics of the time series, there are several issues of importance. It needs to be established whether the statistic indicates a physiological quantity (for example, sleep stage or the occurrence of an epileptic seizure or susceptibility to ventricular fibrillation). This can only be demonstrated by showing that the value of the statistic changes in a consistent manner as some physiological condition changes, or by showing differences between populations in physiologically distinct states. Perhaps the most remarkable claims for the application of dimension analysis have been made by Skinner *et al.* [72, 49], who claim that the onset of ventricular fibrillation can be predicted based in a fall of the pointwise correlation dimension to a value near 1.

If a nonlinear statistic, such as the dimension, distinguishes time series from different physiological conditions, it is important to know if the statistic reflects information apparent visually in the time series or that can be found from more conventional measures such as the autocorrelation function. For example, EEGs are conventionally classified by the energy in various bands of the power spectrum. Do changes in dimensionality of the EEG follow these power spectral changes, or vice versa? Questions such as these can perhaps be addressed by systematic use of surrogate data. ("Theoretical" arguments that a given nonlinear statistic is orthogonal to a conventional statistic such as the power spectrum, need to be examined with care. For example, the use of the zero-crossing of the autocorrelation function to set the embedding lag τ in correlation dimension calculations may introduce a link between the power spectrum and the calculated dimension.)

In many cases the justification for using a given statistic (such as the correlation dimension) is founded on assumptions that may not be appropriate in physiology, such as that the system is deterministic or that all transients have died out, or that the level of noise is small. In these cases the hope is that the statistic will nonetheless prove to have physiological meaning even when the assumptions do not hold. A better approach might be to use statistics that do not make unwarranted assumptions, for example, statistics that are intended to provide useful dynamical information even for stochastic systems. For example, Pincus [64] has introduced an "approximate entropy" statistic that can be interpreted for stochastic systems in terms of Markov chains. Nychka [62] has described an algorithm for calculating local divergence (i.e., Lyapunov exponents) which is based in nonparametric regression and therefore expressly designed to be resistant to small amounts of noise.

The analysis of complex time series requires significant skills in mathematics and computer analysis of data. Since very few physiologists or physicians have such skills, progress in the applications of nonlinear dynamics to physiology and medicine requires interdisciplinary groups. If nonlinear dynamic phenomena turn out to be important in medicine, it will be necessary in the future to offer training in nonlinear mathematics to a subset of physicians. Since complex dynamics cut across all branches of medicine, and it is unlikely that more than a handful of physicians will be interested in strong mathematical training, one can foresee a time when a new medical specialty, a *dynamicist*, will take a place amongst cardiologists, neurologists, and the other "ists" of medicine.

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