# Is Fibrillation Chaos?

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**Ventricular fibrillation is examined to determine whether it is an instance of deterministic chaos. Surface ECGs from dogs in fibrillation were used to generate a state space representation of fibrillation. Our analysis failed to identify a low-dimensional attractor that could be associated with fibrillation. The results suggest that fibrillation is similar to a nonchaotic random signal. We note, however, that such random-looking but nonchaotic behavior can also be generated by a nonlinear deterministic system. (*Circulation Research* 1990;67:886–892)**

Although the heart is a complex biological organ, simple mathematical descriptions of its electrical activity often suffice.[1–10] One type of cardiac electrical activity that does not at first sight seem amenable to simple mathematical description is ventricular fibrillation (VF). In VF, small sections of cardiac muscle contract in an uncoordinated manner,[11] leading rapidly to death. VF has traditionally been described as "turbulent" or "chaotic." Recently, however, it has been suggested that there are fairly regular patterns underlying VF, as evidenced by studies of spatial electrical activity during VF[12] or the narrow power spectrum of ECGs recorded during VF.[13]

The seemingly contradictory finding of order and disorder in VF suggests that VF might be an instance of deterministic chaos. This hypothesis has been strengthened by observations that simple deterministic models of cardiac electrical conduction can show fibrillationlike behavior.[3,10] Furthermore, evidence of a period-doubling in hearts susceptible to fibrillation[14] corresponds to one theoretical description of the transition to chaos from a nonchaotic behavior.

A finding that VF is chaos would suggest that there is a simple mechanism at work in VF and would provide guidance in the search for clinical precursors to VF (e.g., a sequence of period-doublings). We examine the question of whether VF is chaos by attempting to construct a representation of a deterministic system that can duplicate ECGs recorded from VF. The results suggest that fibrillation is not characterized by a low-dimensional dynamical system and thus is not usefully thought of as being chaotic.

Developments in the past decade in the field of nonlinear dynamical systems theory[15–17] have emphasized the existence of mathematical and physical systems that are deterministic yet do not allow long-range predictions to be made accurately and that appear turbulent or disordered. Such systems are called chaotic, particularly when the system can be described in terms of only a few dynamical variables. The behavior of chaotic systems is irregular and random-looking.

Although there is no universally accepted definition of deterministic chaos, it is widely accepted that a chaotic system has three attributes: 1) it is a deterministic dynamical system, 2) it has sensitive dependence on initial conditions, and 3) it has an attractor.

A dynamical system is a system that at each instance of time has a state and a rule that tells how the state changes in time. The state is generally written as a vector of quantities; the set of all such states is called the state space (or phase space). For example, in the case of a simple harmonic oscillator (e.g., a mass on a spring) the state is the instantaneous velocity and position.

In a deterministic dynamical system the rule that describes how the state changes with time can depend only on the state. For each state there is only one possible change in state with time. By plotting out in the state space a succession of instantaneous states, one draws the system's trajectory. In a deterministic system, the trajectory can never cross itself.

Sensitive dependence on initial conditions refers to whether nearby trajectories come closer together or separate. To illustrate, consider two identical systems (i.e., systems with identical rules governing their dynamics) with slightly different initial states. If, as time progresses, the two states drift apart, the system shows sensitive dependence on initial conditions. A trivial example is the dynamics of money in a bank account: two accounts with identical interest rates (i.e., identical dynamic rules) but with different initial deposits will drift apart in their respective worths.

Sensitive dependence on initial conditions is responsible for the difficulty in predicting the future state of chaotic dynamical systems. Any error in estimating the initial or current state of the system will be magnified in future predictions.

An attractor is a region of state space to which all nearby trajectories eventually tend. For instance, the attractor of a damped harmonic oscillator is the single point (velocity=0, position=0). Dynamical systems generally have an attractor when there is some mechanism that takes "energy" out of the system.

When a system has an attractor and sensitive dependence on initial conditions, the dynamics often appear turbulent.* This is because sensitive dependence is pulling apart nearby states (stretching), while the dissipation that gives rise to the attractor is bringing together distant states (folding).

One of the most useful quantities to use in characterizing a chaotic dynamical system is the dimension of its attractor. Because the attractor is a subset of the state space, it can have a smaller dimension than the state space. Generally, the attractors of chaotic dynamical systems have a noninteger dimension; that is, they are fractals. For instance, the Lorenz equations have a three-dimensional state space, while the attractor is of dimension 2.06.[18]

There are several means of calculating the dimension of an attractor. One way, by Grassberger and Procaccia,[18] is to count the number of pairs of points $C(l)$ on the attractor closer together than distance $l$. The dimension is

$$\nu(l) = \frac{d \log C(l)}{d \log l}$$

The dimension ($\nu$) can be considered a function of the length scale ($l$), although for mathematical objects one generally considers the limit as $l \rightarrow 0$.

If a dynamical system is deterministic, the trajectory cannot cross itself in state space. When the state space is dimension 3 or higher, one cannot expect the trajectory actually to intersect itself in any finite length of time. Rather, if the system is not deterministic, different sections of the trajectory will approach one another closely and approximately orthogonally. However, the inevitable presence of noise, either measurement or affecting the system dynamics, destroys the strict determinism of the system. Noise imposes a characteristic length scale on the system (corresponding to the signal/noise ratio) and has the effect of fattening the trajectory from a line into a tube whose radius is that characteristic length scale. Sections of the trajectory that run together through the same tube are indistinguishable and should not be regarded as evidence for the nondeterminism of the idealized, nonnoisy system. The near-crossings of

interest are cases where the tubes come together approximately orthogonally, that is, crossings where the trajectories are distinguishable at first and then become indistinguishable.

To test for the existence of such near-crossings, one constructs a putative state vector of dimension $n$. This dimension is called the embedding dimension. The dimension of the trajectory in the $n$-dimensional state space is calculated. Call this $\nu_n(l)$. Next, the process is repeated for a state vector of higher dimension. For a nondeterministic system where there are many near-crossings in many directions in the putative state space, $\nu_n(l)$ increases with $n$. For a deterministic system, $\nu_n(l)$ saturates at some $n$ (at least for $l$ greater than the length scale imposed by noise) because the trajectory is untangled by the higher-dimensional embedding and at some $n$ is completely untangled. At larger $n$, $\nu_n(l)$ decreases because of the finite length of the signal used to construct the trajectory. The maximum $\nu_n(l)$ (maximizing over $n$) is the dimension of the attractor at length scale $l$. In general, one tries to look for an $l$ at which $\nu(l)$ is constant, that is, a plateau in the graph of $\nu$ versus $l$.

Once it has been established that the system is a deterministic dynamical system, the questions of the existence of an attractor and sensitive dependence on initial conditions can be addressed. The existence of an attractor is demonstrated if the addition of more data does not change the dimension $\nu_n(l)$ and does not cause the trajectory to visit new regions of state space (this presumes that start-up transients have died out by the time the initial data were collected). Sensitive dependence on initial conditions can be established by looking for positive Lyapunov exponents by the method of Wolf et al,[19] for example. Previous work by several groups has applied dimensional analysis to biomedical signals. Babloyantz and Destexhe, Mayer-Kress, and others have examined EEGs,[20,21] vector ECGs,[22] and heart rate.[23] This research has suggested that the dimensional analysis shows a finite dimension for the systems that generate these signals. Some of the difficulties of dimensional calculations in these types of systems are discussed in the literature.[24–26]

## Methods

To test the hypothesis that fibrillation is chaos, we sought to determine whether fibrillation satisfies the three components of the definition of chaos given above. The first step is to construct a dynamical system representation of fibrillation that can then be tested for determinism and the existence of an attractor. Because one generally does not measure the actual state vector of a system, a surrogate state vector must be constructed from measured signals. This can be done by using lagged values of the measured signal, a method that has been theoretically justified by Takens.[27]

The data used in this study were three-lead surface ECGs from dogs in which fibrillation had been

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*Some physicists have informally made a distinction between "chaos" and "turbulence." They call a system turbulent when it is genuinely complicated, involving a large number of degrees of freedom, and chaotic when the system appears irregular but the dynamics are simple, involving only a few degrees of freedom.

induced electrically or in which it had occurred spontaneously. The data were collected in a series of experiments performed by Smith et al.[14] Four episodes, each from a different dog, were studied. The dogs used were in a variety of states, hypothermic or rewarmed from hypothermia, with coronary artery ligations and without. In many cases, fibrillation had been induced one or more times previously and terminated with electric shock. The episodes of VF were uninterrupted by attempts to terminate or modify the fibrillation, and the animals were on mechanical respiration at a fixed rate.

ECGs were collected from three approximately orthogonal leads and recorded in analog form on an FM tape recorder (tape speed, 9.5 cm/sec, 3 dB; cutoff 0–500 Hz). Later, the analog signal was replayed into an analog low-pass filter for antialiasing purposes (360-Hz cutoff, five-pole Butterworth) and digitized (12 bits) at 1,000 Hz. Because there was negligible energy above 30 Hz in the power spectrum of fibrillation, the antialiasing filter would have little effect on the signal of interest, serving only to remove noise. In particular, phase shifts introduced by the analog filter do not affect the signal of interest, because these phase shifts occur only near the filter's cutoff frequency. The digitized data were filtered to remove respiration artifact by use of a 1,027-point, zero phase-shift digital high-pass filter with cutoff at 2 Hz. In all cases, the spectrum of the unfiltered signal had a large peak at respiration frequency (usually 0.25 Hz, corresponding to 15 breaths/min) and very little spectral content between 1 and 4 Hz. While this filtering cannot remove respiratory influence on fibrillation, it does remove the obvious respiratory artifact from the ECG.

The power spectrum of the ECG from the episodes of fibrillation we examined is consistent with that found by Goldberger et al[13] and Herbschleb et al,[28] namely, a fairly narrow peak (see Figure 5).

For each episode of VF, several signal segments were generated from an 8-second segment of signal starting 1–2 seconds after the change from a tachycardialike rhythm to fibrillation. The signal segments ranged from 2 to 8 seconds at sampling rates of 62.5, 125, 250, and 1,000 Hz. The reduced sampling rates were achieved by digital decimation (i.e., taking every *n*th point). No additional antialiasing filtering was done; however, the power spectrum of the original signals showed negligible energy above 30 Hz.

With the method of lags, a 21-dimensional vector signal was constructed. This vector signal consisted, at each time point, of the original three-channel ECG and six lagged values of the three-channel ECG separated by $\tau$. The calculations were repeated for three different $\tau$ values ranging from 30 to 100 msec (total window length, 180–600 msec). A common technique for establishing a minimum value for $\tau$ uses the first zero-crossing of the autocorrelation function, in this case approximately 30 msec. However, because the envelope of the autocorrelation function decays slowly, it is appropriate to investigate larger
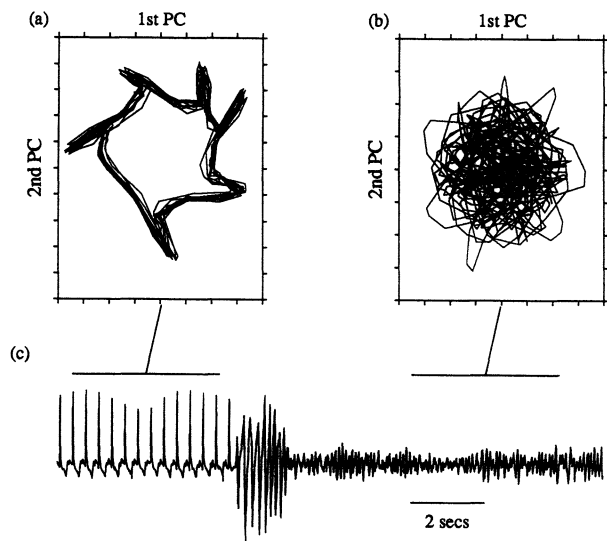


FIGURE 1.    *Projections onto two dimensions of the trajectory drawn in a 21-dimensional embedding. The two dimensions chosen correspond to the two largest principal components (PCs). Panel a: The trajectory of 4 seconds of atrially paced rhythm. Panel b: The trajectory of 4 seconds of fibrillation. Panel c: One lead of the three-lead ECG from which the data plotted in panels a and b were taken. The segments used are indicated by the horizontal line segments.*

values of $\tau$. Another technique for setting $\tau$ makes use of the mutual information in the time-delayed versions of the signal.[29] Because the results were substantially independent of $\tau$, we report here the results only for $\tau=50$ msec.

The method of principal components[30] was used to select smaller-dimensional subsets from the 21-dimensional vector. To make an *n*-dimensional putative state vector, the *n* largest principal components of the cloud of points in the 21-dimensional space were selected. The trajectory was determined, and the dimension of the trajectory was calculated for $n=5$, 9, 13, and 21. When calculating the dimension for segment lengths less than 8 seconds, an average was taken of $C(l)$ from several overlapping segments that fit within the overall 8-second segment.

Altogether, then, 144 different dimension calculations were done on each fibrillation episode (4 sampling rates×3 segment lengths×3 lag values×4 state space dimensions).

Finally, for each of the four sampling rates and three segment lengths, a random signal was generated that had an indentical power spectrum to the original. The dimension calculations were repeated on the randomized signals for $\tau=50$ and $n=5$, 9, 13, and 21. All calculations were performed on a SUN 3/60 workstation (Sun Microsystems, Mountain View, Calif.).

## Results

Figure 1b shows the slice of phase space corresponding to the two largest principal components of a 4-second segment of fibrillation. The figure illus-
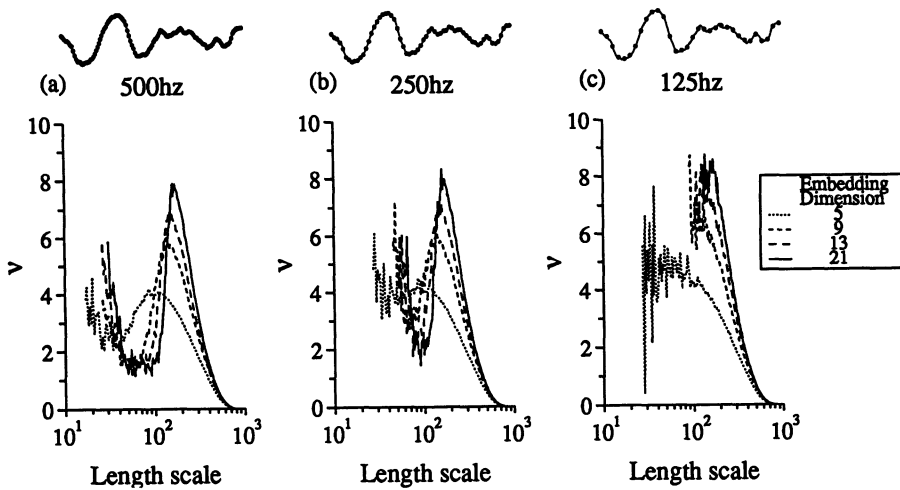
FIGURE 2. *Dimension as a function of length scale* $[\nu(1)]$ *for one of the episodes of fibrillation. An 8-second segment was used, and* $\nu(1)$ *is shown at three sampling rates. Above each graph, a short segment (0.25 second) of the signal is marked to indicate the particular samples at each sampling rate.*

trates that a two-dimensional state is not sufficient to represent fibrillation as a deterministic system: the trajectory is highly tangled. In contrast, the state space plot of an atrially paced, nonfibrillatory rhythm (Figure 1a) suggests that the dynamics of normal rhythms are deterministic.

Rather than relying on pictorial representations of the system trajectories, which are restricted to two dimensions, the $\nu_n(l)$ calculations allow high-dimensional state spaces to be investigated. Figure 2a illustrates the results of the dimension calculations for one episode of VF, with a segment length of 8 seconds, $\tau=50$ msec, and a sampling rate of 500 Hz. The graph shown is representative of all the episodes studied, at all $\tau$ values. At length scales of approximately 10–30, there is a plateau with $\nu_n \approx 1$. In contrast, Figure 2c shows $\nu_n(l)$ when the sampling rate is 125 Hz. The plateau has disappeared, and $\nu_\nu(l)$ increases with $n$ for just about any $l$ chosen.

Figure 3 summarizes the results from all four episodes of VF for the different segment lengths. Each symbol in the plot corresponds to a $\nu_n(l)$ curve; the $l$ used was chosen by eye to be that at which $\nu_n(l)$ appeared to "level out." The error bars correspond to the estimated uncertainty in the estimate of $\nu_n$. Mayer-Kress[24] has proposed a method for estimating a dimension from a $\nu_n(l)$ curve; he suggests finding

the region of that curve best fit by a straight line of a certain length. Although this technique works well when a system has a well-defined dimension, it underestimates the dimension of noisy signals.

Figure 4 presents the difference between the dimension of each VF episode and the corresponding randomized signal. A negative value means that the randomized signal has a higher dimension than the VF episode.

## Discussion

There is much imprecision and reliance on judgment that goes into the selection of a length scale and the estimate of a dimension for a signal. Unfortunately, there is little theory to guide the objective choice of a specific length scale of importance.

Nonetheless, one artifact of the calculation can be pointed out and avoided: the plateau at $\nu \approx 1$ in Figure 2a is not an attribute of VF, but of the high sampling rate of 500 Hz. In traditional signal processing, sampling at unnecessarily high rates (above the Nyquist frequency) merely adds redundancy to the signal. In the case of dimensionality calculations, however, oversampling biases the calculations toward low dimension. This occurs because, at too high sampling rates, the sampled points on the trajectory are like closely spaced beads on a string. At small
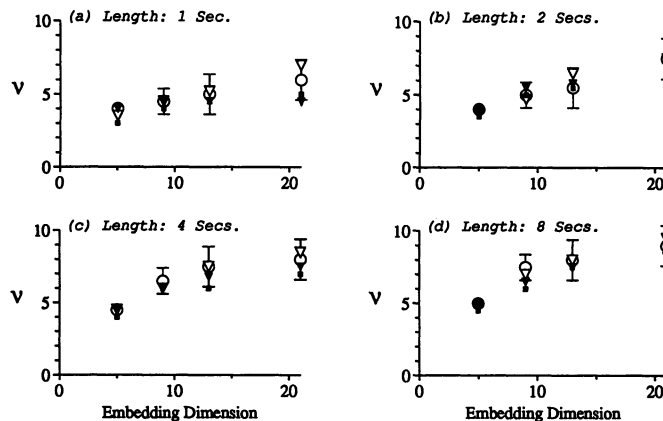


FIGURE 3. *The estimated dimensions (v) found for various embedding dimensions with various segment rates as indicated. Each episode of fibrillation is plotted as a different symbol.*
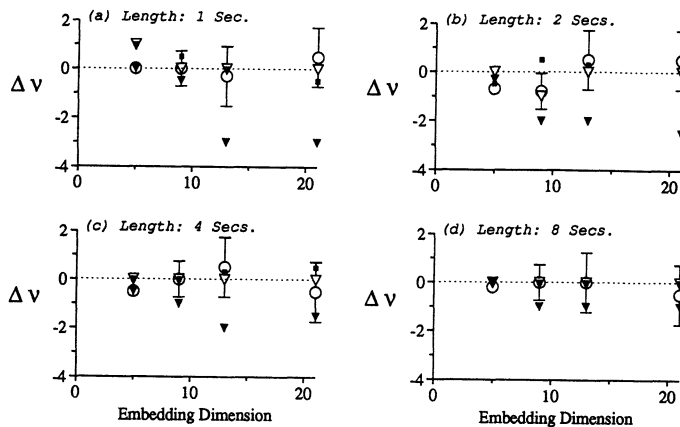
FIGURE 4. *The difference between the dimension estimate ($\Delta v$) for each episode of fibrillation and the corresponding randomized signal.*

length scales only the one-dimensional structure of a single strand is evident; the overall structure of the attractor is lost. The following standard can be applied to determine whether a signal is oversampled: if it is predominantly the case that nearest neighbor points in state space are points that were sampled consecutively, the trajectory is oversampled.

When oversampling is compensated for by decimation, the dimensionality results in Figures 3 and 4 suggest that a low-dimensional deterministic system cannot be constructed to describe the episodes of VF studied here and that there is no significant difference between VF and a randomized signal with the same power spectrum.

One important issue is that of stationarity: the condition of the heart is gradually deteriorating during VF. We attempted to deal with this problem by using short segments of data. However, we cannot rule out the possibility that the state of the heart is changing even during the 2-second data segments used. The use of even shorter segments would not solve the problem, because the dimension calculations become unreliable for very short segments.[24]

It is possible that a high-dimensional deterministic description of fibrillation does exist. However, determining reliably that a high-dimensional attractor does exist takes a considerable amount of data; one

rule of thumb is that for an $m$-dimensional attractor, $10^m$ points are needed. Figure 3 suggests that $m$ might be no less than 8. At 125 Hz, more than 200 hours of VF data would be required!

To verify the validity of the methods, we used a deterministic chaotic system to generate a signal similar to fibrillation (see Figure 5). As our control deterministic chaotic system, we used the Lorenz equations.[31] We integrated numerically the three differential equations of the Lorenz system, setting the time scale so that the resulting signal would have a peak at approximately 10 Hz. The $z$-component of the Lorenz system has a pattern that is superficially similar to a fibrillatory ECG. To enhance this similarity, we bandpass filtered between 5 and 16 Hz (using a 127-point finite impulse response filter). The power spectrum of the resulting signal, which we will refer to as $z(t)$, and a portion of the signal are shown in Figure 5.

Because the Lorenz equations are completely deterministic and three dimensional, the dimension of their trajectory can be no larger than 3. The bandpass filtering may obscure the shape of the attractor, but it can be expected that the bandpass-filtered signal should have a dimension between 2 and 3.

We sampled $z(t)$ at 100 Hz and took segments of 1, 2, 4, and 8 seconds. Each of these segments was
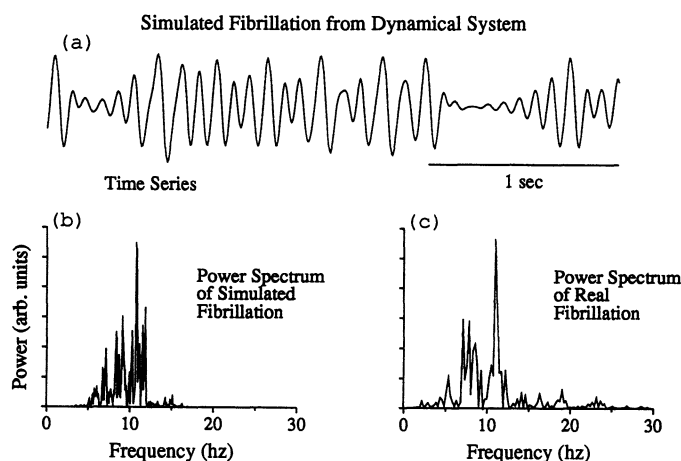


FIGURE 5. *A portion of a fibrillationlike signal synthesized from a chaotic deterministic dynamical system. The power spectrum of this signal and the power spectrum of a representative episode of fibrillation are shown.*

TABLE 1. Estimated Dimensions of Simulated Fibrillation Signals

| Signal (seconds) | z(t) | $z_r(t)$ | zz(t) | $zz_r(t)$ |
|---|---|---|---|---|
| 1 | 4.5±1.5 | 4.0±1.5 | 4.0±1.5 | 3.0±1.5 |
| 2 | 3.5±1.0 | 5.0±1.0 | 4.0±1.0 | 4.5±1.0 |
| 4 | 3.0±0.7 | 6.0±0.7 | 4.5±0.7 | 6.0±0.7 |
| 8 | 2.5±0.5 | 6.5±0.5 | 5.0±0.5 | 6.5±0.5 |

z(t), Power spectrum; $z_r(t)$, phase-randomized version of z(t); zz(t), vector signal; $zz_r(t)$, phase-randomized version of zz(t).

individually subjected to a dimensionality calculation with a maximum embedding dimension of 21 and a maximum window length of 500 msec. The same analysis was done on a phase-randomized version of z(t), called $z_r(t)$.

In addition, we repeated the analysis on a vector signal, which we will refer to as zz(t), consisting of two independent realizations (with slightly different parameters) of the bandpass-filtered z-component of the Lorenz equations. This system should theoretically have a dimension between 4 and 6. The phase-randomized version is denoted $zz_r(t)$. The results are shown in Table 1 for an embedding dimension of 21.

For the low-dimensional dynamical system represented by z(t), the method is easily able to distinguish between the deterministic signal and the randomized signal for signal lengths of 4 seconds or greater. The distinction is present but less marked between the higher-dimensional signal zz(t) and the corresponding randomized signal. The results suggest that to look for a low-dimensional attractor of dimension 2–3, at least 4 seconds of data should be analyzed. For an attractor of dimension between 4 and 6, at least 8 seconds of data should be used.

These results are derived for signals with a spectrum similar to dog fibrillation that has a spectral peak near 10 Hz. For human fibrillation, the spectral peak of which is near 6 Hz, the necessary segment lengths would be correspondingly longer.

To address the question of whether nonstationarity of the heart during VF was strongly affecting our results, we examined the dimensionality of simulated ECGs from a simple deterministic finite-element model of cardiac electrical conduction, which mimics fibrillation. The model has been described elsewhere.[3,32] The model used was a two-dimensional array of 4,000 hexagonal elements arranged into a cylinder with circumference of 80 elements. An element fires when stimulated by a neighbor's firing, after which it remains refractory for a preassigned time. The refractory periods were spatially inhomogeneous, with a uniform distribution of mean 34 time steps and half-width of 18 time steps. The model was stimulated every 34 time steps for 1,000 time steps, and then stimulation was stopped. After 9,000 more time steps were allowed to elapse, simulated ECGs were collected.

The model was run for several thousand refractory cycles (i.e., the time it takes for a cell to fire, recover,

and fire again), corresponding to approximately 10 minutes of dog VF. The dimensionality results for the model data were very similar to those of real VF; in particular, $v_n(l)$ did not saturate even for embedding dimensions up to 200. However, the model tended, after such long times, to stop fibrillating spontaneously. This means, at least in the model, that fibrillation is not an attractor, but rather a transient. Crutchfield and Kaneko[33] have pointed out other examples of spatially distributed systems that undergo long transients and have labeled such behavior quasistationary transients.

The fibrillatory ECG signals analyzed here provide no evidence that fibrillation reflects a dynamical system with a low-dimensional attractor. The dimensionality calculations are largely consistent with fibrillatory ECGs being random white noise passed through a coloring filter. This does not necessarily imply, however, that fibrillation is not deterministic; the finite-element model of cardiac conduction provides an example of a deterministic dynamical system that can generate random-looking, yet nonchaotic, behavior.

The results presented here may appear to contradict earlier findings by Goldberger et al[13] that fibrillation is an orderly phenomenon. These findings were based on observations of a narrow-band power spectrum for fibrillatory ECGs for 1-second segments. While the power spectra of the episodes of fibrillation we studied were similar to those reported by Goldberger et al[13] and Herbschleb et al,[28] our analysis implicitly includes consideration of the phases as well as amplitudes of the spectral components of fibrillation. That is, our analysis indicates that despite the narrow power spectrum of fibrillation, it is disordered.

In contrast, prefibrillatory ECGs have a dimension near 1: they are highly ordered. The comparative order of prefibrillatory signals is apparent in Figure 1a.

So, is fibrillation chaos? There is little evidence from dimensionality calculations to support the contention that fibrillation is chaos. If fibrillation is chaos, it appears to arise from a high-dimensional system. Because the numerical tools used to study experimental chaotic systems are practical to use only for low-dimensional systems, we conclude that there is little utility in classifying fibrillation as chaotic.

## Acknowledgments

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